CDI-II Prática 22/3/21

$$\begin{tikzcd}
\mathbb{R}^n \arrow[r, "g"', "\text{dif. em } a"'] \arrow[rr, bend right, "f\circ g"'] & \mathbb{R}^p \arrow[r, "f"', "\text{dif. em } g(a)"'] & \mathbb{R}^m
\end{tikzcd}$$

$$a \longmapsto g(a) \longmapsto f(g(a))$$

$f\circ g$ dif. em a e

$$\boxed{\underset{m\times n}{D(f\circ g)(a)} = \underset{m\times p}{Df(g(a))}\ \underset{p\times n}{Dg(a)}}$$

Derivada da composta.

1- $D(f \circ g)(1,1)$ $\qquad a = (1,1)$

3×2 $\qquad g(a) = g(1,1)$

$\quad \|$ $\qquad \boxed{g(1,1) = (1,0)}$

$Df(g(1,1))\, Dg(1,1)$

$= Df(1,0)\; Dg(1,1)$

$3 \times 2 \qquad 2 \times 2$

$\frac{d}{dx} \arctan(u(x))$

$= \frac{u'(x)}{1+u(x)^2}$

etc.

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2- σ = "sigma" $\qquad \sigma(t) = F(\gamma(t))$

γ = "gama"

$$
\begin{tikzcd}
t \arrow[r, mapsto] & \gamma(t) \arrow[r, mapsto] & F(\gamma(t)) \\
\mathbb{R} \arrow[r] \arrow[rr, bend right, "\sigma = F \circ \gamma"'] & \mathbb{R}^3 \arrow[r] & \mathbb{R}^1
\end{tikzcd}
$$

$$\sigma(t) = F(\gamma(t))$$

$$D\sigma(t) = \underbrace{\sigma'(t)}_{1\times 1} = \underbrace{DF(\gamma(t))}_{1\times 3}\,\underbrace{D\gamma(t)}_{3\times 1}$$

$F : \mathbb{R}^n \to \mathbb{R}$, $\underbrace{DF(a)}_{\text{matriz}} = \underbrace{\nabla F(a)}_{\text{vector}} =$ gradiente de F em a.

$$F(x,y,z) = x^2 + y^2 + z^2 + 1$$

$$DF(x,y,z) = \begin{bmatrix} 2x & 2y & 2z \end{bmatrix}$$

$$DF(\gamma(t)) = DF(\operatorname{sen} t, t^2, \cos t)$$

$$= \begin{bmatrix} 2\operatorname{sen} t & 2t^2 & 2\cos t \end{bmatrix}.$$

$$D\gamma(t) = \begin{bmatrix} \cos t \\ 2t \\ -\operatorname{sen} t \end{bmatrix}$$

$$\sigma'(t) = \begin{bmatrix} 2\operatorname{sen} t & 2t^2 & 2\cos t \end{bmatrix} \begin{bmatrix} \cos t \\ 2t \\ -\operatorname{sen} t \end{bmatrix}$$

$$= 4t^3.$$

Regra de cadeia.

$$F(x,y,z) \qquad (x,y,z) = \gamma(t)$$
$$= (x(t), y(t), z(t))$$

$$\sigma(t) = F(\gamma(t)) = F(x(t), y(t), z(t))$$

$$\sigma'(t) = \frac{\partial F}{\partial x}\frac{dx}{dt} + \frac{\partial F}{\partial y}\frac{dy}{dt} + \frac{\partial F}{\partial z}\frac{dz}{dt}$$

(t)

$(x(t), y(t), z(t))$

$$t \longmapsto (x(t), y(t), z(t)) \longmapsto F(\gamma(t))$$

$$t \longmapsto \underbrace{(x(t), y(t), z(t))}_{\gamma(t)} \longmapsto F(\gamma(t)) = F(x(t), y(t), z(t))$$

$$\sigma'(t) = 2x(t)\frac{dx}{dt}(t) + 2y(t)\frac{dy}{dt}(t) + 2z(t)\frac{dz}{dt}(t)$$

$$\begin{aligned}
x(t) &= \operatorname{sen} t &&\longrightarrow \frac{dx}{dt}(t) = \cos t \\
y(t) &= t^2 &&\longrightarrow \frac{dy}{dt}(t) = 2t \\
z(t) &= \cos t &&\longrightarrow \frac{dz}{dt}(t) = -\operatorname{sen} t
\end{aligned}$$

$$\sigma'(t) = 4t^3$$

3 — $f \circ g$

$g: \mathbb{R}^2 \to \mathbb{R}^3$

$f: \mathbb{R}^3 \to \mathbb{R}$

$$\mathbb{R}^2 \xrightarrow{g} \mathbb{R}^3 \xrightarrow{f} \mathbb{R}$$

$$(0,0) \longmapsto g(0,0) \longmapsto f(g(0,0)) = f(0,1,2)$$

$$g(0,0) = (0,1,2)$$

$$\underbrace{D(f\circ g)(0,0)}_{1\times 2} = Df(g(0,0))\, Dg(0,0)$$

$$= \underbrace{Df(0,1,2)}_{1\times 3}\, \underbrace{Dg(0,0)}_{3\times 2}$$

$$= \underbrace{\begin{bmatrix} ye^x + z^2 & e^x & 2xz \end{bmatrix}}_{(0,1,2)} \begin{bmatrix} 0 & 1 \\ 2 & 3 \\ 4 & 0 \end{bmatrix} = \begin{bmatrix} 2 & 8 \end{bmatrix}$$

$$5 \quad 1 \quad 0$$

$$D_v(f\circ g)(0,0) = D(f\circ g)(0,0)\cdot v$$

$$v = (1,2)$$

$$= \begin{bmatrix} 2 & 8 \end{bmatrix}\begin{bmatrix} 1 \\ 2 \end{bmatrix} = 18$$

4 — $\sigma(x) = f(\operatorname{sen} x,\ x + e^{x})$

$$\underset{\mathbb{R}}{x} \xmapsto{\ g\ } \underbrace{\overset{\mathbb{R}^2}{(\operatorname{sen} x,\ x + e^{x})}}_{g(x)} \longmapsto \overset{\mathbb{R}^3}{f(g(x))}$$

$g(0) = (0,1)$

$$\sigma'(0) = Df(g(0))\,Dg(0)$$

$$Dg(x) = \begin{bmatrix} \cos x \\ 1 + e^{x} \end{bmatrix} \qquad g(x) = (\sin x, x + e^{x})$$

$$Dg(0) = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$$

$$\sigma'(0) = Df(g(0))\, Dg(0)$$

$$= Df(0,1)\, Dg(0)$$

$$= \begin{bmatrix} 1 & 0 \\ 2 & 1 \\ 3 & 2 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \end{bmatrix}$$

$$= \begin{bmatrix} 1 \\ 4 \\ 7 \end{bmatrix}$$

5-a) $(x,y,z) \xmapsto{f} f(x,y,z) \xmapsto{g} g(f(x,y,z))$

$$\begin{tikzcd} \mathbb{R}^3 \arrow[r] \arrow[rr, bend right, "g \circ f"'] & \mathbb{R}^3 \arrow[r] & \mathbb{R} \end{tikzcd}$$

$$D(g\circ f)(1,1,0) = Dg(f(1,1,0))\, Df(1,1,0)$$

1×3 ? $\quad 1\times 3 \quad 3\times 3$

$$\begin{bmatrix} \cdot & \cdot & \cdot \end{bmatrix} = \begin{bmatrix} -1 & 0 & 3 \end{bmatrix} \begin{bmatrix} \cdot & 2y & \cdot \\ \cdot & 1 & \cdot \\ \cdot & e^z & \cdot \end{bmatrix}$$

$\begin{matrix} 2 \\ 1 \\ 1 \end{matrix}$

$(1,1,0)$

$-2+0+3 = 1$

$f(1,1,0) = (2,2,1)$ $\qquad$ $Dg(2,2,1)$

$Dg(2,2,1) = \begin{bmatrix} -1 & 0 & 3 \end{bmatrix} = (-1,0,3)$

Usar regra da cadeia
função $g : \mathbb{R}^3 \to \mathbb{R}$.

$$\underbrace{g\big(f(x,y,z)\big)}_{h(x,y,z)} = g\big(u(x,y,z), v(x,y,z), w(x,y,z)\big)$$

$$(x,y,z) \longmapsto \big(u(x,y,z), v(x,y,z), w(x,y,z)\big) \longrightarrow g(\dots)$$

$$\nabla g(2,2,1) = Dg(2,2,1)$$

$$= \left[\frac{\partial g}{\partial u} \;\; \frac{\partial g}{\partial v} \;\; \frac{\partial g}{\partial w}\right]_{(2,2,1)}$$

$$\frac{\partial h}{\partial y}(1,1,0) = \frac{\partial g}{\partial u}(2,2,1)\frac{\partial u}{\partial y}(1,1,0) + $$
$$+ \frac{\partial g}{\partial v}(2,2,1)\frac{\partial v}{\partial y}(1,1,0) + $$
$$+ \frac{\partial g}{\partial w}(2,2,1)\frac{\partial w}{\partial y}(1,1,0)$$

5-b) é diferente de 5a) porque g é dada em vez de Dg.

$$Dg(u,v,w) = \begin{bmatrix} 2u & -2v & e^{w} \end{bmatrix}$$

— $h = g \circ f$

$$\frac{\partial h}{\partial z} = \frac{\partial g}{\partial u}\frac{\partial u}{\partial z} + \frac{\partial g}{\partial v}\frac{\partial v}{\partial z} + \frac{\partial g}{\partial w}\frac{\partial w}{\partial z}$$

etc.

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7- $F(x, y, \overbrace{g(x,y)}^{z}) = 0$

$\Longleftarrow \quad z = g(x,y)$

$$\frac{\partial F}{\partial z}(x,y,z) \neq 0$$

$$F(x, y, z(x,y)) = 0$$

$$z(x,y) = g(x,y) \qquad \frac{\partial g}{\partial x}, \quad \frac{\partial g}{\partial y}$$

R. cadeic

$$\underset{y}{\frac{\partial F}{\partial x}}(x,y,z(x,y)) + \underbrace{\frac{\partial F}{\partial z}(x,y,g(x,y))}_{\neq 0} \underset{y}{\frac{\partial z}{\partial x}}(x,y)$$

$$\underset{y}{\frac{\partial z}{\partial x}}(x,y) \equiv \underset{y}{\frac{\partial g}{\partial x}}(x,y) = - \frac{\underset{y}{\frac{\partial F}{\partial x}}(x,y,z(x,y))}{\frac{\partial F}{\partial z}(x,y,z(x,y))}$$

$$F\Big(\underset{\substack{\| \\ x}}{u(x,y)},\ \underset{\substack{\| \\ y}}{v(x,y)},\ \underset{\substack{\| \\ g(x,y)}}{w(x,y)}\Big)$$

6- Regra de cadeia.

$$g\Big(\underbrace{g(x^2, xy, x+y)+e^{x}}_{u(x,y)},\ \underbrace{xy}_{v(x,y)},\ \underbrace{g(x,x,x)}_{w(x,y)}\Big)$$

$$= g\big(u(x,y), v(x,y), w(x,y)\big)$$

$$u(x,y) = g\big(a(x,y), b(x,y), c(x,y)\big) + e^{x}$$